

The Value of G was first Determined by?

Gravitation

❖ Claudius Ptolemy proposed 'Geocentric Theory'. According to this theory all planets including sun are revolving round the earth.

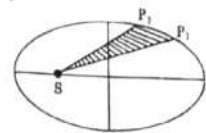
❖ Copernicus proposed 'Heliocentric theory'. According to this theory all planets including earth are revolving round the sun.

❖ **Kepler's Law of Planetary Motion:**

i) **I law :** Every planet revolves round the sun in an elliptical orbit, with the sun lying at one focus of the ellipse. This is also called law of orbits.

ii) **II Law:** The areal velocity of a planet round the sun is constant or every planet sweeps out equal areas in equal intervals of time.

Or, $\frac{dA}{dt} = \text{constant}$. Where 'dA' is the area swept out by the planet in a time 'dt'.



iii) The areal velocity is $\frac{dA}{dt} = \frac{1}{2} r^2 \omega = \frac{1}{2} Rv$.
 $\frac{dA}{dt} = \frac{1}{2} \frac{mvr^2\omega}{m} = \frac{1}{2} \frac{L}{2m} = \frac{L}{4m}$

- From the above equation L is constant. Therefore, Kepler's second law is a direct consequence of the law of conservation of angular momentum.
- According to Kepler's second law, when a planet is closest to the sun its speed is maximum and it is farthest from the sun its speed is minimum.
 $L = mvr = \text{constant}$
 $vr = \text{constant}$ $v_1 r_1 = v_2 r_2$
 If v_1, v_2 are the velocities of a planet when it is at distances r_1, r_2 from the sun respectively.

Kepler's third law (or) law of period:

- According to Kepler's third law the square of the period of revolution (T) of a planet round the sun is directly proportional to the cube of the semi major axis (R) of the elliptical orbit.
 $\therefore T^2 \propto R^3$
- According to Kepler's third law as the distance of the planet from the sun increase, duration of the year of the planet increase.

Newton's Universal Law of Gravitation :

- Every body in universe attracts every other body with a force which is directly proportional to the product of the masses of the

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two bodies and inversely proportional to the square of the distance between them.

$$F = G \frac{m_1 m_2}{d^2}$$

- Where ' m_1 ' and ' m_2 ' are the masses of two bodies and 'd' is the distance between their centers and 'G' is the universal gravitational constant.

i) The universal gravitational constant 'G' is the numerically equal to the force of attraction between two bodies of unit masses, separated by unit distance.

ii) The value of $G = 6.67 \times 10^{-8} \frac{\text{dyne-cm}^2}{\text{gm}^2}$ (C.G.S. system)
 $= 6.67 \times 10^{-11} \frac{\text{newton-m}^2}{\text{kg}^2}$ (M.K.S system)
 The dimensional formula of G is $[M^{-1} L^3 T^{-2}]$

iv) The value of G was first determined by Henry Cavendish.

v) This force acts along the line joining the centers of the masses.

vi) The gravitational force between two bodies is independent of the presence of the third body and the medium in which the bodies are situated.

vii) The gravitational force between two bodies is action - reaction pair.

viii) Gravitational force is conservative force work done by it is independent of the path.

❖ **Gravitational Intensity:**

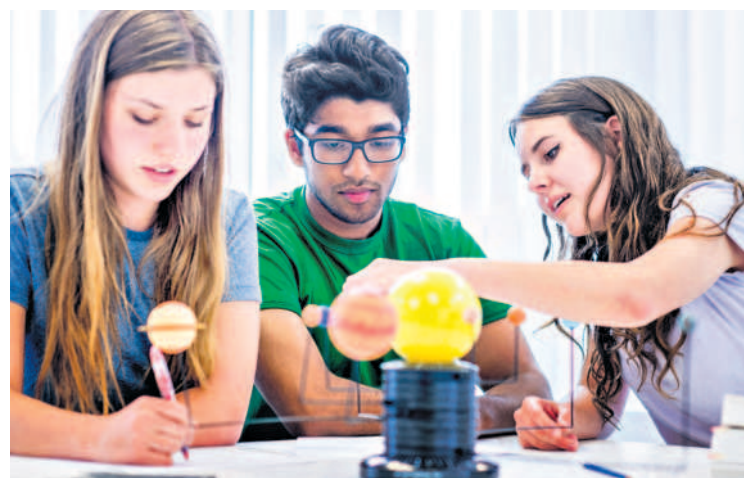
- It is the gravitational force of attraction exerted by a body on unit mass. Gravitational intensity due to a mass 'M' at a distance 'r' from the center of mass is given by $E_g = \frac{GM}{r^2}$
- It is a vector quantity expressed in the units newton - kg^{-1} .

❖ **The gravitational force between two bodies are action - reaction pair.**

- If two bodies of masses m_1 and m_2 are separated by a distance d then the distance of the point where intensity of gravitational field is zero from m_1 is $x = \frac{d}{\sqrt{\frac{m_2}{m_1} + 1}}$

❖ **Gravitational Potential:**

- The amount of work done in bringing a unit mass from infinity to that point is called



gravitational potential at the point.

- Gravitational Potential due to a mass 'M' at a distance 'r' is given by $V_G = -\frac{GM}{r}$.
- Gravitational Potential is a scalar quantity.
- Gravitational Potential is measured in J. kg^{-1} .
- Dimensional formula of gravitational potential is $L^2 T^{-2}$.

❖ **Gravitational Potential energy of a two body system :**

- The amount of work done in bringing the two bodies from infinity separation to the given separation is stored as potential energy.
- If ' m_1 ', ' m_2 ' are two masses separated by infinite distance the gravitational potential energy of the system when they are brought to a separation of 'r' is given by $U = -G \frac{m_1 m_2}{r}$
- [The negative sign indicates that the two bodies are attracting each other].
- If three particles of masses ' m_1 ', ' m_2 ' and ' m_3 ' are kept at three corners of an equilateral triangle of side 'd' then gravitational P.E. of the system is given by $U = -\frac{G}{d} (m_1 m_2 + m_2 m_3 + m_3 m_1)$
- If a body is kept on surface of the earth then its gravitational P.E. = $-\frac{GMm}{R}$
- Where M = mass of the earth
 R = radius of the earth
- If a body is at a height h above surface of the earth then its gravitational P.E. = $-\frac{GMm}{R+h}$
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- If $h \ll R$ then $W = mgh$
- If a body is projected with

velocity v from surface of the earth so that it goes upto height nR from surface of the earth then $v = \sqrt{\frac{2ynR}{1+n}}$

❖ **Acceleration due to gravity:**
 The acceleration acquired by a freely falling body due to gravitational pull is called acceleration due to gravity. It is denoted by 'g'. 'g' is a vector quantity.

Units of 'g' :

(i) cm/sec^2 (C.G.S. system)
 (ii) m/sec^2 (S.I. system)

- The dimensional formula of 'g' is $[M^0 L^1 T^{-2}]$.
- The value of 'g' near the surface of the earth is equal to $9.8 \text{ m}/\text{sec}^2$.
- The value of 'g' maximum as poles is $(9.83 \text{ m}/\text{sec}^2)$ and minimum at equator $9.78 \text{ m}/\text{sec}^2$.
- The value of 'g' is numerically equal to the force experienced by unit mass placed in the gravitational field, i.e., $g = GM/R^2$ where 'M' is the mass of the earth and 'R' is the radius of the earth.

Note: The mass of the earth can be estimated using the following formula $M = gR^2/G$. Its value is nearly equal to $6 \times 10^{24} \text{ kg}$.

i) If M_1, M_2 are the masses and R_1, R_2 are the radii of two planets then $\frac{g_1}{g_2} = \frac{M_1}{M_2} \times \frac{R_2^2}{R_1^2}$

- The acceleration due to gravity (g) on a planet of radius R and density d is given by $g = \frac{4}{3} \pi GRd$.
- If R_1, R_2 are radii and d_1, d_2 are the densities of two planets $\frac{g_1}{g_2} = \frac{R_1 d_1}{R_2 d_2}$

iii) If a body is dropped from same height on two different planets then

1) Striking Velocity $v \propto \sqrt{g}$
 $\therefore \frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2}}$

2) Time of descent $t \propto \frac{1}{\sqrt{g}}$
 $\therefore \frac{t_1}{t_2} = \sqrt{\frac{g_2}{g_1}}$

iv) If a body weight on surface of earth is 'W' kg then its weight on surface of planet of mass M_1 , radius R_1 , density d_1 is given by $W_1 = W \left[\frac{m_1}{m} \times \frac{R^2}{R_1^2} \right] = W \left[\frac{R_1 d_1}{Rd} \right]$

Where M = mass of the earth; R = radius of the earth; d = density of the earth

- The lines joining the places on the earth having same values of 'g' are called isogams.
- Gravity meter and ETVOS gravity balance are used to measure changes in acceleration due to gravity.

❖ **Variation of 'g':**

a) **Effect of altitude :** If 'g' is the acceleration due to gravity on the surface of the earth 'g' is the acceleration due to gravity at the height h above the surface of the earth.
 $g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(R+h)^2}$
 Hence, $\frac{g'}{g} = \frac{R^2}{(R+h)^2}$

i) For small values of h, $g' = g \left(1 - \frac{2h}{R} \right)$ Thus as height increases the value of 'g' decreases.

ii) At height h, $\frac{dg}{g} \times 100 = \frac{2h}{R} \times 100$

- The acceleration due to gravity becomes x % of its value on surface of the earth at height $= R \left[\frac{10}{\sqrt{x}} - 1 \right]$
- The height where acceleration due to gravity is 1/x of that on surface of the earth $h = R(\sqrt{x} - 1)$
- Effect of depth:** If 'g' is acceleration due to gravity at the surface of the earth, 'g₁' the acceleration due to gravity at a depth 'd' below the surface of the earth.
 $\frac{g_1}{g} = \left(1 - \frac{d}{R} \right)$
 $\therefore g_1 = g \left(1 - \frac{d}{R} \right)$

Thus as the depth 'd' increase, the acceleration due to gravity decreases.

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