

The Value of G was first Determined by?

Gravitation

❖ Claudius Ptolemy proposed 'Geocentric Theory'. According to this theory all planets including sun are revolving round the earth.

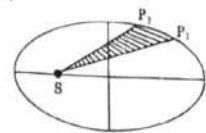
❖ Copernicus proposed 'Heliocentric theory'. According to this theory all planets including earth are revolving round the sun.

❖ **Kepler's Law of Planetary Motion:**

i) **I law :** Every planet revolves round the sun in an elliptical orbit, with the sun lying at one focus of the ellipse. This is also called law of orbits.

ii) **II Law:** The areal velocity of a planet round the sun is constant or every planet sweeps out equal areas in equal intervals of time.

Or, $\frac{dA}{dt} = \text{constant}$. Where 'dA' is the area swept out by the planet in a time 'dt'.



iii) The areal velocity is $\frac{dA}{dt} = \frac{1}{2} r^2 \omega = \frac{1}{2} Rv$.
 $\frac{dA}{dt} = \frac{1}{2} \frac{mvr^2\omega}{m} = \frac{1}{2} \frac{L}{2m} = \frac{L}{4m}$

- From the above equation L is constant. Therefore, Kepler's second law is a direct consequence of the law of conservation of angular momentum.
- According to Kepler's second law, when a planet is closest to the sun its speed is maximum and it is farthest from the sun its speed is minimum.
 $L = mvr = \text{constant}$
 $vr = \text{constant}$ $v_1 r_1 = v_2 r_2$
 If v_1, v_2 are the velocities of a planet when it is at distances r_1, r_2 from the sun respectively.

Kepler's third law (or) law of period:

- According to Kepler's third law the square of the period of revolution (T) of a planet round the sun is directly proportional to the cube of the semi major axis (R) of the elliptical orbit.
 $\therefore T^2 \propto R^3$
- According to Kepler's third law as the distance of the planet from the sun increase, duration of the year of the planet increase.

Newton's Universal Law of Gravitation :

- Every body in universe attracts every other body with a force which is directly proportional to the product of the masses of the

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two bodies and inversely proportional to the square of the distance between them.

$$F = G \frac{m_1 m_2}{d^2}$$

- Where ' m_1 ' and ' m_2 ' are the masses of two bodies and 'd' is the distance between their centers and 'G' is the universal gravitational constant.
- The universal gravitational constant 'G' is the numerically equal to the force of attraction between two bodies of unit masses, separated by unit distance.
- The value of $G = 6.67 \times 10^{-8} \frac{\text{dyne-cm}^2}{\text{gm}^2}$ (C.G.S. system)
 $= 6.67 \times 10^{-11} \frac{\text{newton-m}^2}{\text{kg}^2}$ (M.K.S system)
 The dimensional formula of G is $[M^{-1} L^3 T^{-2}]$
- The value of G was first determined by Henry Cavendish.
- This force acts along the line joining the centers of the masses.
- The gravitational force between two bodies is independent of the presence of the third body and the medium in which the bodies are situated.
- The gravitational force between two bodies is action - reaction pair.
- Gravitational force is conservative force work done by it is independent of the path.

❖ **Gravitational Intensity:**

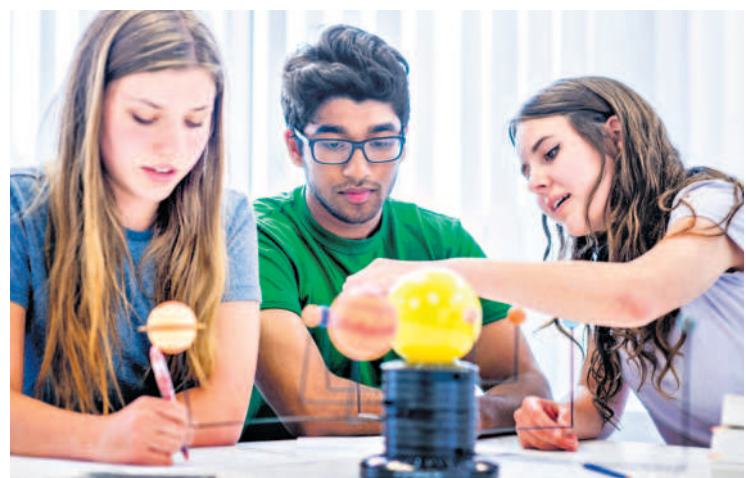
- It is the gravitational force of attraction exerted by a body on unit mass. Gravitational intensity due to a mass 'M' at a distance 'r' from the center of mass is given by $E_g = \frac{GM}{r^2}$
- It is a vector quantity expressed in the units newton - kg^{-1} .

❖ **The gravitational force between two bodies are action - reaction pair.**

- If two bodies of masses m_1 and m_2 are separated by a distance d then the distance of the point where intensity of gravitational field is zero from m_1 is $x = \frac{d}{\sqrt{\frac{m_2}{m_1} + 1}}$

❖ **Gravitational Potential:**

- The amount of work done in bringing a unit mass from infinity to that point is called



gravitational potential at the point.

- Gravitational Potential due to a mass 'M' at a distance 'r' is given by $V_G = -\frac{GM}{r}$.
- Gravitational Potential is a scalar quantity.
- Gravitational Potential is measured in J. kg^{-1} .
- Dimensional formula of gravitational potential is $L^2 T^{-2}$.

❖ **Gravitational Potential energy of a two body system :**

- The amount of work done in bringing the two bodies from infinity separation to the given separation is stored as potential energy.
- If ' m_1 ', ' m_2 ' are two masses separated by infinite distance the gravitational potential energy of the system when they are brought to a separation of 'r' is given by $U = -G \frac{m_1 m_2}{r}$
- [The negative sign indicates that the two bodies are attracting each other].
- If three particles of masses ' m_1 ', ' m_2 ' and ' m_3 ' are kept at three corners of an equilateral triangle of side 'd' then gravitational P.E. of the system is given by $U = -\frac{G}{d} (m_1 m_2 + m_2 m_3 + m_3 m_1)$
- If a body is kept on surface of the earth then its gravitational P.E. = $-\frac{GMm}{R}$
- Where M = mass of the earth
 R = radius of the earth
- If a body is at a height h above surface of the earth then its gravitational P.E. = $-\frac{GMm}{R+h}$
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- If $h \ll R$ then $W = mgh$
- If a body is projected with

velocity v from surface of the earth so that it goes upto height nR from surface of the earth then $v = \sqrt{\frac{2ynR}{1+n}}$

❖ **Acceleration due to gravity:**
 The acceleration acquired by a freely falling body due to gravitational pull is called acceleration due to gravity. It is denoted by 'g'. 'g' is a vector quantity.

Units of 'g' :

(i) cm/sec^2 (C.G.S. system)
 (ii) m/sec^2 (S.I. system)

- The dimensional formula of 'g' is $[M^0 L^1 T^{-2}]$.
- The value of 'g' near the surface of the earth is equal to 9.8 m/sec^2 .
- The value of 'g' maximum as poles is (9.83 m/sec^2) and minimum at equator 9.78 m/sec^2 .
- The value of 'g' is numerically equal to the force experienced by unit mass placed in the gravitational field, i.e., $g = GM/R^2$ where 'M' is the mass of the earth and 'R' is the radius of the earth.

Note: The mass of the earth can be estimated using the following formula $M = gR^2/G$. Its value is nearly equal to $6 \times 10^{24} \text{ kg}$.

i) If M_1, M_2 are the masses and R_1, R_2 are the radii of two planets then $\frac{g_1}{g_2} = \frac{M_1}{M_2} \times \frac{R_2^2}{R_1^2}$

- The acceleration due to gravity (g) on a planet of radius R and density d is given by $g = \frac{4}{3} \pi GRd$.
- If R_1, R_2 are radii and d_1, d_2 are the densities of two planets $\frac{g_1}{g_2} = \frac{R_1 d_1}{R_2 d_2}$

iii) If a body is dropped from same height on two different planets then

1) Striking Velocity $v \propto \sqrt{g}$
 $\therefore \frac{v_1}{v_2} = \sqrt{\frac{g_1}{g_2}}$

2) Time of descent $t \propto \frac{1}{\sqrt{g}}$
 $\therefore \frac{t_1}{t_2} = \sqrt{\frac{g_2}{g_1}}$

iv) If a body weight on surface of earth is 'W' kg then its weight on surface of planet of mass M_1 , radius R_1 , density d_1 is given by $W_1 = W \left[\frac{m_1}{m} \times \frac{R^2}{R_1^2} \right] = W \left[\frac{R_1 d_1}{Rd} \right]$

Where M = mass of the earth; R = radius of the earth; d = density of the earth

- The lines joining the places on the earth having same values of 'g' are called isogams.
- Gravity meter and ETVOS gravity balance are used to measure changes in acceleration due to gravity.

❖ **Variation of 'g':**

a) **Effect of altitude :** If 'g' is the acceleration due to gravity on the surface of the earth 'g' is the acceleration due to gravity at the height h above the surface of the earth.
 $g = \frac{GM}{R^2}$ and $g' = \frac{GM}{(R+h)^2}$
 Hence, $\frac{g'}{g} = \frac{R^2}{(R+h)^2}$

i) For small values of h, $g' = g \left(1 - \frac{2h}{R} \right)$ Thus as height increases the value of 'g' decreases.

ii) At height h, $\frac{dg}{g} \times 100 = \frac{2h}{R} \times 100$

- The acceleration due to gravity becomes x % of its value on surface of the earth at height $= R \left[\frac{10}{\sqrt{x}} - 1 \right]$
- The height where acceleration due to gravity is 1/x of that on surface of the earth $h = R(\sqrt{x} - 1)$
- Effect of depth:** If 'g' is acceleration due to gravity at the surface of the earth, 'g1' the acceleration due to gravity at a depth 'd' below the surface of the earth.
 $\frac{g'}{g} = \left(1 - \frac{d}{R} \right)$
 $\therefore g' = g \left(1 - \frac{d}{R} \right)$

Thus as the depth 'd' increase, the acceleration due to gravity decreases.

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