నమస్తే తెలంగాణ
నిపుణ II
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పోటీపరీక్షల ప్రత్యేకం
ఆదివారం 15 అక్టోబర్ 2023

నేను వచ్చాను, నేనే చూశాను, నేను జయించాను అని నినాదం ఇచ్చిన వారు?
జూలియస్ సీజర్

అసాధ్యం మూర్ఖుల నిఘంటువులో మాత్రమే కనిపించే పదం అని పేర్కొన్నది?
నెపోలియన్

నేనే విప్లవాన్ని, నేనే విప్లవ శిశువుని అని అన్నది ఎవరు?
నెపోలియన్

ప్రపంచ కార్మికులారా ఏకం కండి అని నినాదం ఇచ్చినది ఎవరు?
కారల్‌మార్క్స్

The Radius of the Geostationary Orbit?

Gravitation

continued from Oct 13th

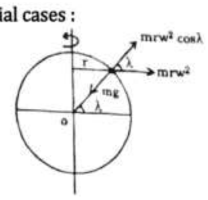
- The value of acceleration due to gravity at the center of the earth g = 0.
- If change in value of 'g' at height h above the surface of earth same as depth 'd' below the earth surface then d = 2h.
- The decrease in value of 'g' at depth d = Δg =gd/R.
- The fractional change in value of g = Δg/g = d/R
- The depth at which acceleration due to gravity is 1/x of that on surface of the earth d = R ((x-1)/x)

Note : If g if the acceleration due to gravity on surface of the earth and g1 is the acceleration due to gravity at height h (h<<R) above the surface of earth is

given by $g^{11} = \frac{g+g^1}{2}$.

- **Effect of rotation of earth:**Due to the rotation of the earth, the value of acceleration due to gravity g1at a given place is given by
  $g^1 = g – \omega^2 R \cos^2\lambda$
- where ω is the angular velocity, R is the radius of the earth and λis the latitude of the place.

Special cases :



i) At the poles $\lambda = 90^0$
$\therefore g^1 = g - \omega^2 R (0)^2$
$(\cos \lambda = 0^0)$

i.e., The value of g is maximum at the poles.

At the equator λ = 00

$\therefore g^1 = g - \omega^2 R (1)^2$ (cos λ =00)

$\therefore g^1 = g – \omega^2 R$

- i.e., The value of g is minimum at the equator.
- The value of 'g' at poles does not depend on the speed of rotation of the earth. But at equator the value of 'g' decreases with the increase of speed of rotation of the earth.
- If the speed of rotation of the earth increases the length of the day decreases.
- If earth suddenly stops its rotation then (a) 'g' at poles remains constant. (b) 'g' at equator increases by $R\omega^2$ (0.034m/s²)
- The value of 'g' at equator is zero when
- The earth rotates 17 times faster
- The angular velocity of

  earth $\omega = \sqrt{\frac{g}{R}}$

  $= 1.25 \times 10^{-3}$ rad/s

  (c) The time period of earth

  $T = 2s \sqrt{\frac{R}{g}} = 84.6$min.

❖ **Geostationary orbit or Parking orbit:**

- An orbit in which the time period of revolution is exactly equal to 24 hours is called geostationary orbit.
- The plane of the orbit of a geostationary satellite should pass through Geo –equatorial plane there appears.

a) The relative velocity of geostationary satellite with respect to the earth is zero.

b) To be stationary with respect to the earth.

c) The radius of the geostationary orbit = 42000 km (nearly).

d) The height of the geostationary satellite above the surface of the earth = 36000 km.

e) The speed of a geostationary satellite = 3 km s^{-1}.

f) The angular velocity of a geostationary satellite

$(\omega) = \frac{2\pi}{T} = \frac{2\pi}{86,400}$ radian / sec.

g) The direction of revolution of geostationary satellite should be same as the direction of reaction of earth (i.e., from west to east).

❖ **Orbital Velocity (V0):**

The orbital velocity of a satellite orbiting near the surface of the earth is given by

$V_0 = \sqrt{gR} = \sqrt{\frac{GM}{R}}$

❖ The orbital angular velocity of a satellite $\omega^0 = \sqrt{\frac{g}{R}} = \sqrt{\frac{GM}{R^3}}$

❖ Orbital velocity decreases with the increase of height.

❖ The orbital velocity depends on the mass and radius of the planet.

❖ Orbital velocity for earth bound satellites = 7.92 km/sec.

- Orbital velocity for moon bound satellite = 1.7 km/s.

❖ **Escape Velocity (V_e):**

- Every body of mass 'm' is bound to the earth with some (mgR). If we supply kinetic energy greater than this (mgR) energy the body will escape from the earth's influence. The minimum velocity with which a body must be projected such that it should escape from the earth's influence and never return to the earth is known as escape from the earth's influence and never return to the earth is known as escape velocity.

PHYSICS

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This is given by

$V_e = \sqrt{2gR} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{8}{3}\pi GR^2 d}$

i) Its value does not depend on the direction of projection.

ii) Escape velocity is independent of the mass , size, shape of the projectile.

iii) Its value V_e = 11.2 km/sec (on the surface of the earth).

iv) its value V_e = 2.5 km/sec (on the surface of the moon).

∴ Moon has no atmosphere because Ve value is small.

v) If V_1 and V_2 are the escape velocities on two different planets then

$\frac{V_1}{V_2} = \sqrt{\frac{M_1 R_2}{M_2 R_1}} = \sqrt{\frac{g_1 R_2}{g_2 R_1}} = \sqrt{\frac{d_1 R_2^2}{d_2 R_1^2}}$

vi) If a body is projected with a velocity V (V > V_e) then velocity of the body at infinity $= \sqrt{V^2 - V_e^2}$

❖ **Relation between Escape velocity and Orbital velocity:**

Escape velocity

$(V_e) = \sqrt{2}$ x orbital velocity

$\Rightarrow V_e = \sqrt{2}\, V_o = 1.414\, V_o$

- If orbital velocity of a body is increased √2 times or its K.E is doubled (i.e., its speed is increased by 41.4%) the body attains escape velocity and escapes into space.

Nature of Orbits:

If the critical velocity of projection is

$Ve = \sqrt{gR} = \sqrt{\frac{GM}{R}}$ then the body is projected with a horizontal velocity V.

- If V < V_o, the body falls to the ground.
- If V = V_o the body rotates in a circular orbit.
- If V_o< V < Ve, the body revolves in a elliptical orbit.
- If V = V_e the body just escapes from the gravitational field along a parabolic path.
- If V > V_e the body escapes along a hyperbolic path.

❖ **For satellite:**

i) The orbital velocity of the satellite in orbit at a height is given by $V_o = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}}$

ii) The orbital angular velocity of the satellite is given by

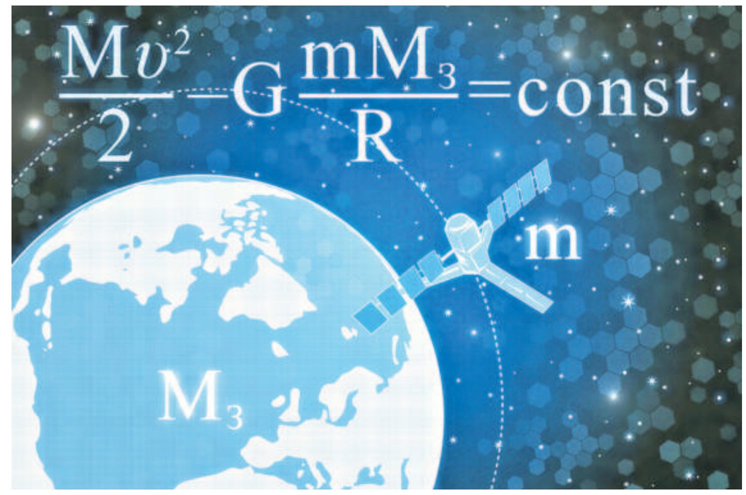
$\omega = \frac{V_0}{R+h} = \sqrt{\frac{GM}{(R+h)^3}}$

iii) The time period of the satellite is given by $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

iv) If h < < R (surface satellite)

a) $V_o = \sqrt{\frac{GM}{R}} = \sqrt{gR}$

b) $\omega = \frac{V_0}{R} = \sqrt{\frac{GM}{R^3}}$



c) $T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} = 84.6$ min.

v) **Energies of the satellite:**

a) P.E of the satellite $= -\frac{GMm}{R+h}$

If h < < R then

$P.E = -\frac{GMm}{R+h} = -mgR.$

b) K.E of the satellite $= \frac{GMm}{2(R+h)}$.

If h < < R then

$K.E = \frac{GMm}{2R} = \frac{mgR}{2}$

c) T.E of the satellite = P.E + K.E

$= -\frac{GMm}{2(R+h)}$ If h < < R then

$T.E = -\frac{GMm}{2R} = - mgR/2$

- The negative sign indicate that the satellite is bound of the earth.
  d) P.E = 2 K.E
  e) Binding energy = - T.E . = mgR/2.
- For a satellite in the orbit P.E. is greater than K.E. (numerically)
- When the total energy of satellite becomes equal to zero it escapes into space in a parabolic path.
- If the polar ice caps melt, the duration of the day increase.
- When a satellite is shifted from a lower orbit to a higher orbit its P.E. increase and K.E. decreases.
- If a satellite of mass m is launched in circular orbit at height h from surface of the earth, then work done. (or) required energy = mgR² [1/R-1/2(R+h)].

Inertial mass: Inertial mass of a body is the ratio of net force acting on the body to acceleration produces in the body. It is given by m = F/a.

Gravitational Mass: Gravitational mass of a body is the ratio of gravitational force acting on the body to the acceleration due to gravity of the body. It is given by m = W/g.

Worked Examples :

- If the force of attraction between two spheres of masses 40kg and 80kg respectively kept at a distance of 50cm F1 , (find the force of attraction between them when the distance between them is doubled.) two bodies.

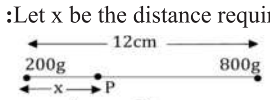
Sol : Given $F_1 = \frac{Gm_1m_2}{d^2}$; $d^1 = 2d$

$F = \frac{Gm_1m_2}{(d)^2} = \frac{Gn_1n_2}{(2d)^2} = \frac{Gm_1m_2}{4d^2}$

$= \frac{F_1}{4}$

- Masses of 200gm and 800gm are 12cm apart. At which point from the 200gm mass, the intensity of the gravitational field due to the two masses would be zero?

Sol : Let x be the distance required.



$x = \frac{d}{\sqrt{\frac{m_2}{m_1}}+1} = \frac{12}{\sqrt{\frac{800}{200}}+1} = 4cm$

- If there is a planet with the same mass and radius half of that of the earth, what is the acceleration due to gravity on the surface of the planet if it is g on the surface of the earth?

Sol : On the surface of the earth

$\frac{GM}{R^2}$ (1)

On the planet,

$g_1 = \frac{GM}{\left[\frac{R^2}{2}\right]} = \frac{GM \times 4}{R^2}$ ... (2)

From (1) and (2), $\frac{g^1}{g} = 4$

$\frac{g'}{g} = \frac{GM4}{R^2} \times \frac{R^2}{4M}$

$\therefore g' = 4g.$

❖ A body of 200kg. wt. is on the earth's surface. Find its weight at a place 6400km above the surface of the earth (radius of the earth is 6400km).

Sol : On the surface of the earth,

W = mg

At a height h above the surface of the earth, W =mg'

$\therefore \frac{W'}{W} = \frac{g'}{g}$, $g = \frac{GM}{R^2}$ and $g' = \frac{GM4}{(R+h)^2}$

$\frac{W'}{W} = \frac{R^2}{(R+n)^2}$

$\frac{W'}{200} = \frac{6400^2}{4 \times 6400^2} = \frac{1}{4}$

$\frac{W'}{200} = \frac{1}{4}$ $W' = 50kg$

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