

# Velocity of the First Ball after Collision?

## COLLISIONS

**Collision:** Collision is an interaction between two or more bodies in which sudden changes of momenta takes place.

- The time duration of collision is very small.
- During collision the two colliding bodies may or may not come into physical contact.
- If the colliding bodies move along a straight line joining their center of mass before and after collision such a collision is called one dimensional or head on collision.
- If the two colliding bodies, move in a plane before and after collision such a collision is called two-dimensional collision.

**Law of Conservation of Linear Momentum:** Law of conservation of linear momentum states that when no external force acts on a system, the total momentum of the system remains constant both magnitude and direction.

**Types of Collisions:**  
**Elastic Collision:** The Collision in which both momentum and kinetic energy are conserved is called an elastic collision.

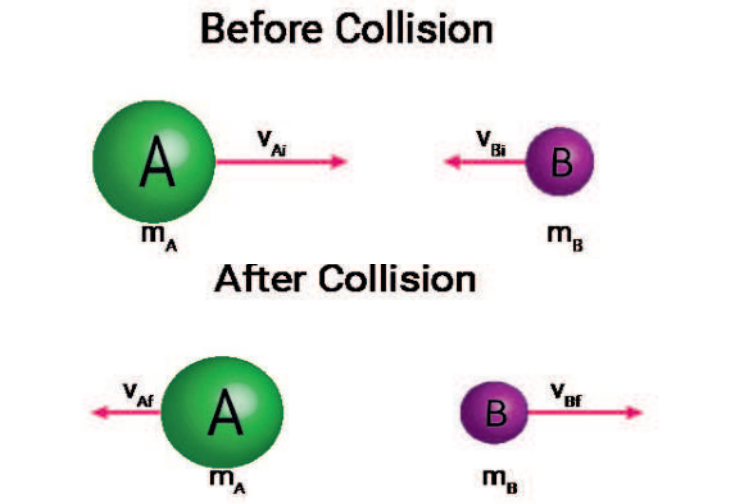
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{ and } \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

**Ex:** Collision between ivory balls, atomic particles etc.

ii) **Inelastic Collision:** The collision in which only the momentum is conserved is called as inelastic collision. i.e.,  $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

**Ex:** Collision between two balls of putty, collision between a bullet and a wooden block... etc.

- In one dimensional elastic collision. The relative velocity of approach before collision is equal to the relative velocity of separation after collision. i.e.,  $u_1 - u_2 = v_2 - v_1$  This is known as Newton's formula
- When the body of masses  $m_1$  and  $m_2$  moving in the same direction along a straight line with velocities  $u_1, u_2$  collide with each other and  $v_1, v_2$  are their velocities after collision (if the collision is elastic) then



Velocity of the first ball after collision

$$v_1 = \left[ \frac{m_1 - m_2}{m_1 + m_2} \right] u_1 + \left[ \frac{2m_2}{m_1 + m_2} \right] u_2$$

Velocity of the second ball after collision

$$v_2 = \left[ \frac{2m_1}{m_1 + m_2} \right] u_1 + \left[ \frac{m_2 - m_1}{m_1 + m_2} \right] u_2$$

- When a heavy body collides with a light body at rest and the collision is perfectly elastic, the heavy body continues to move with the same velocity where as the light body moves with a velocity equal to double the velocity of the heavy body.  $M_1 \gg m_2 \therefore v_2 = 2u_1$   
 $v_1 = u_1$
- When a light body collides with a heavy body at rest and the collision is perfectly elastic, the light body rebounds with the same velocity whereas heavy body remains at rest.  $M_1 \ll m_2 \therefore v_1 = u_1, v_2 = 0$
- When two bodies of equal masses moving in opposite directions with same speed collide, if the collision is elastic each body rebounds with same speed after collision.
- In the case of perfectly elastic collision if the second body is at rest before collision then the velocities of the bodies after collision are

$$v_1 = \left[ \frac{m_1 - m_2}{m_1 + m_2} \right] u_1; v_2 = \left[ \frac{2m_1}{m_1 + m_2} \right] u_1; \frac{v_1}{v_2} = \left[ \frac{m_1 - m_2}{2m_1} \right]$$

**Note:** In the perfectly elastic collision there is no loss of KE of the system but KE of one body is transferred to another body.

- When a body of mass " $m_1$ " moving with kinetic energy " $E_1$ " undergoes perfectly elastic collision with another body of mass " $m_2$ " which is at rest.
- The amount of KE transferred from  $m_1$  to  $m_2$  is

- The fraction of KE transferred from  $m_1$  to  $m_2$   $\frac{E_2}{E_1} = \frac{4m_1m_2}{(m_1+m_2)^2}$
- The percentage of KE transferred from  $m_1$  to  $m_2$  is  $\frac{E_2}{E_1} \times 100 = \frac{4m_1m_2}{(m_1+m_2)^2} \times 100$

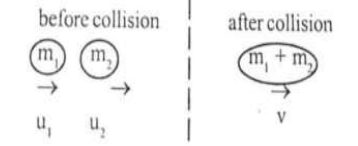
- When two bodies of equal masses suffering one dimensional elastic collision. They simply exchange their velocities after collision  $\therefore v_1 = u_2; v_2 = u_1$
- When a body collide with another body of same mass at rest after collision, the first body comes to rest whereas the second body moves with velocity of the first body.  $\therefore v_1 = 0; v_2 = u_1$

### Perfectly inelastic collision :

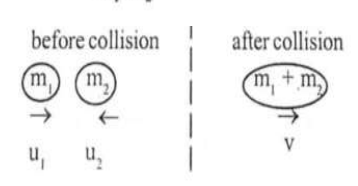
- In this collision two bodies colaces after collision and moves with common velocity.
- Let two bodies of masses  $m_1, m_2$  be moving with the velocity  $u_1$  and  $u_2$  in the same direction and undergoes perfectly inelastic collision, there the common velocity after collision

$$(v) = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

- If the bodies moves in opposite direction before collision then



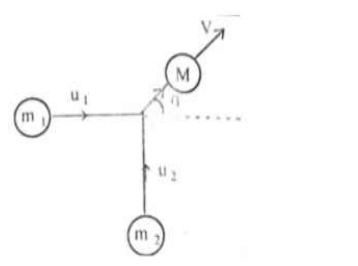
If the second body is at rest before collision then

$$v = \frac{m_1u_1}{m_1 + m_2}$$


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- When the bodies of mass  $m_1$  and  $m_2$  be moving along position
- x-axis and along positive y-axis with the velocity  $u_1$  and  $u_2$  respectively and undergoes perfectly inelastic collision then the common velocity after the collision

$$v = \frac{\sqrt{(m_1u_1)^2 + (m_2u_2)^2}}{M} \text{ where } M = m_1 + m_2 \text{ and } \tan \theta = \frac{p_2}{p_1} = \frac{m_2u_2}{m_1u_1} \therefore \theta = \tan^{-1} \left( \frac{m_2u_2}{m_1u_1} \right)$$



**Note:** In the case of perfectly inelastic collision the system loses some KE in the form of heat energy.

- When a body of mass " $m_1$ " moving with the velocity " $u_1$ " undergoes perfectly inelastic collision with another object of mass " $m_2$ " which is moving in the same direction with the velocity " $u_2$ " then the loss in KE is

$$\Delta k = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} [u_1 - u_2]^2 \text{ (minimum)}$$

- If the second body moves opposite direction then loss in K.E is

$$\Delta k = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} [u_1 + u_2]^2 \text{ (maximum)}$$

- If the second object is at rest before the collision the loss in K.E is

$$\Delta k = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} [u_1]^2$$

- When a body of mass " $m_1$ " moving with the K.E ' $E$ ' undergoes perfectly inelastic collision with another body of mass  $m_2$  which is at rest then loss

$$\text{in K.E is given by } \Delta k E = E \left[ \frac{m_2}{m_1+m_2} \right]$$

- Fraction of loss in K.E is given by  $\frac{\Delta k E}{k E} = \frac{m_2}{m_1+m_2}$
- Percentage of loss in K.E is given by  $\frac{\Delta k E}{k E} \times 100 = \left[ \frac{m_2}{m_1+m_2} \right] 100$

- Remaining K.E is given by  $\Delta k E^1 = E - \Delta k = E \left[ \frac{m_1}{m_1+m_2} \right]$
- Fraction of loss in K.E is given by  $\frac{\Delta k E}{k E} = \frac{m_2}{m_1+m_2}$
- Percentage of loss in K.E is given by  $\frac{\Delta k E}{k E} \times 100 = \left[ \frac{m_2}{m_1+m_2} \right] 100$
- Remaining K.E is given by  $\Delta k E^1 = E - \Delta k = E \left[ \frac{m_1}{m_1+m_2} \right]$
- Fraction of remaining K.E is given by  $\frac{\Delta k E^1}{k E} = \frac{m_1}{m_1+m_2}$
- Percentage of remaining K.E is given by  $\frac{\Delta k E^1}{k E} \times 100 = \frac{m}{m_1+m_2}$
- The remaining K.E is shared between " $m_1$ " and " $m_2$ " in the direct ratio of their mass.
- The K.E of  $m_1$  after collision is given by  $k E_1^1 = E \left[ \frac{m_1}{m_1+m_2} \right]^2$
- The K.E of  $m_2$  after collision  $k E_2^1 = E \left[ \frac{m_2}{m_1+m_2} \right]^2$

- Semielastic collision (or) Inelastic collision :** Let a body of mass  $m_1$  moving with the velocity  $u_1$  under goes collision with another body of mass  $m_2$  moving with the velocity  $u_2$  in the same direction. The coefficient of restitution is " $e$ ". The velocity of the bodies after the collision is

$$v_1 = \left( \frac{m_1 - em_2}{m_1 + m_2} \right) u_1 + \left( \frac{m_2 + em_2}{m_1 + m_2} \right) u_2$$

$$v_2 = \left( \frac{m_1 + em_1}{m_1 + m_2} \right) u_1 + \left( \frac{m_2 - em_1}{m_1 + m_2} \right) u_2$$

If the second body is at rest before collision  $v_1 = \left( \frac{m_1 - em_2}{m_1 + m_2} \right) u_1$

$$v_2 = \left( \frac{m_1 - em_1}{m_1 + m_2} \right) u_1$$

$$\therefore \frac{v_1}{v_2} = \left( \frac{m_1 - em_2}{m_1 + em_1} \right)$$

- If the mass of the two bodies are equal  $V_1 = \left( \frac{1-e}{2} \right) u_1 + \left( \frac{1+e}{2} \right) u_2$

$$V_2 = \left( \frac{1+e}{2} \right) u_1 + \left( \frac{1-e}{2} \right) u_2$$

- If second body is at rest before collision then

$$V_1 = \left( \frac{1-e}{2} \right) u_1; V_2 = \left( \frac{1+e}{2} \right) u_1$$

$$\therefore \frac{V_1}{V_2} = \left( \frac{1-e}{1+e} \right)$$

- If second body is at rest before collision then

$$K.E. = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} (1-e^2) (u_2 - u_1)^2$$

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