

Velocity of the First Ball after Collision?

COLLISIONS

Collision: Collision is an interaction between two or more bodies in which sudden changes of momenta takes place.

- The time duration of collision is very small.
- During collision the two colliding bodies may or may not come into physical contact.
- If the colliding bodies move along a straight line joining their center of mass before and after collision such a collision is called one dimensional or head on collision.
- If the two colliding bodies, move in a plane before and after collision such a collision is called two-dimensional collision.

Law of Conservation of Linear Momentum: Law of conservation of linear momentum states that when no external force acts on a system, the total momentum of the system remains constant both magnitude and direction.

Types of Collisions:
Elastic Collision: The Collision in which both momentum and kinetic energy are conserved is called an elastic collision.

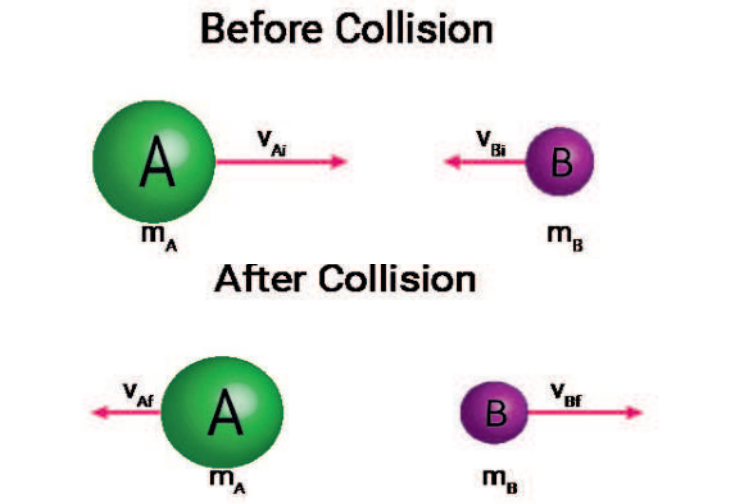
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \text{ and } \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

Ex: Collision between ivory balls, atomic particles etc.

ii) **Inelastic Collision:** The collision in which only the momentum is conserved is called as inelastic collision. i.e., $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

Ex: Collision between two balls of putty, collision between a bullet and a wooden block... etc.

- In one dimensional elastic collision. The relative velocity of approach before collision is equal to the relative velocity of separation after collision. i.e., $u_1 - u_2 = v_2 - v_1$ This is known as Newton's formula
- When the body of masses m_1 and m_2 moving in the same direction along a straight line with velocities u_1, u_2 collide with each other and v_1, v_2 are their velocities after collision (if the collision is elastic) then



Velocity of the first ball after collision
 $v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2} \right] u_1 + \left[\frac{2m_2}{m_1 + m_2} \right] u_2$
 Velocity of the second ball after collision
 $v_2 = \left[\frac{2m_1}{m_1 + m_2} \right] u_1 + \left[\frac{m_2 - m_1}{m_1 + m_2} \right] u_2$

- When a heavy body collides with a light body at rest and the collision is perfectly elastic, the heavy body continues to move with the same velocity where as the light body moves with a velocity equal to double the velocity of the heavy body. $M_1 \gg m_2 \therefore v_2 = 2u_1$
 $v_1 = u_1$
- When a light body collides with a heavy body at rest and the collision is perfectly elastic, the light body rebounds with the same velocity whereas heavy body remains at rest. $M_1 \ll m_2 \therefore v_1 = u_1, v_2 = 0$
- ii) When two bodies of equal masses moving in opposite directions with same speed collide, if the collision is elastic each body rebounds with same speed after collision.
- iii) In the case of perfectly elastic collision if the second body is at rest before collision then the velocities of the bodies after collision are

$$v_1 = \left[\frac{m_1 - m_2}{m_1 + m_2} \right] u_1; v_2 = \left[\frac{2m_1}{m_1 + m_2} \right] u_1; \frac{v_1}{v_2} = \left[\frac{m_1 - m_2}{2m_1} \right]$$

Note: In the perfectly elastic collision there is no loss of KE of the system but KE of one body is transferred to another body.

- When a body of mass " m_1 " moving with kinetic energy " E_1 " undergoes perfectly elastic collision with another body of mass " m_2 " which is at rest.
- The amount of KE transferred from m_1 to m_2 is

$$E_2^1 = E_1 \frac{4m_1m_2}{(m_1+m_2)^2}$$

ii) The fraction of KE transferred from m_1 to m_2 $\frac{E_2^1}{E_1} = \frac{4m_1m_2}{(m_1+m_2)^2}$

iii) The percentage of KE transferred from m_1 to m_2 is $\frac{E_2^1}{E_1} \times 100 = \frac{4m_1m_2}{(m_1+m_2)^2} \times 100$

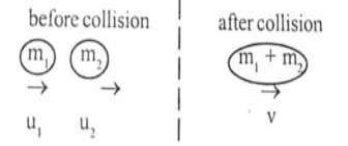
- When two bodies of equal masses suffering one dimensional elastic collision. They simply exchange their velocities after collision $\therefore v_1 = u_2; v_2 = u_1$
- When a body collide with another body of same mass at rest after collision, the first body comes to rest whereas the second body moves with velocity of the first body. $\therefore v_1 = 0; v_2 = u_1$

Perfectly inelastic collision :

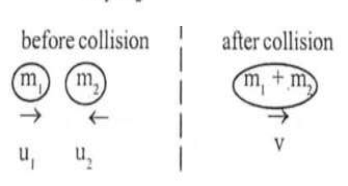
- In this collision two bodies colaces after collision and moves with common velocity.
- i) Let two bodies of masses m_1, m_2 be moving with the velocity u_1 and u_2 in the same direction and undergoes perfectly inelastic collision, there the common velocity after collision

$$(v) = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}$$

- ii) If the bodies moves in opposite direction before collision then



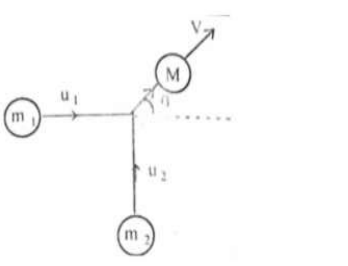
If the second body is at rest before collision then $v = \frac{m_1u_1}{m_1 + m_2}$



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- iii) When the bodies of mass m_1 and m_2 be moving along position
 - x-axis and along positive y-axis with the velocity u_1 and u_2 respectively and undergoes perfectly inelastic collision then the common velocity after the collision

$$v = \frac{\sqrt{(m_1u_1)^2 + (m_2u_2)^2}}{M} \text{ where } M = m_1 + m_2 \text{ and } \tan \theta = \frac{p_2}{p_1} = \frac{m_2u_2}{m_1u_1} \therefore \theta = \tan^{-1} \left(\frac{m_2u_2}{m_1u_1} \right)$$



Note: In the case of perfectly inelastic collision the system loses some KE in the form of heat energy.

- When a body of mass " m_1 " moving with the velocity " u_1 " undergoes perfectly inelastic collision with another object of mass " m_2 " which is moving in the same direction with the velocity " u_2 " then the loss in KE is

$$\Delta k = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} [u_1 - u_2]^2 \text{ (minimum)}$$

- If the second body moves opposite direction then loss in K.E is

$$\Delta k = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} [u_1 + u_2]^2 \text{ (maximum)}$$

- If the second object is at rest before the collision the loss in K.E is

$$\Delta k = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} [u_1]^2$$

- When a body of mass " m_1 " moving with the K.E ' E ' undergoes perfectly inelastic collision with another body of mass m_2 which is at rest then loss

in K.E is given by $\Delta k E = E \left[\frac{m_2}{m_1+m_2} \right]$

- i) Fraction of loss in K.E is given by $\frac{\Delta k E}{k E} = \frac{m_2}{m_1+m_2}$
- ii) Percentage of loss in K.E is given by $\frac{\Delta k E}{k E} \times 100 = \left[\frac{m_2}{m_1+m_2} \right] 100$

- iii) Remaining K.E is given by $\Delta k E^1 = E - \Delta k = E \left[\frac{m_1}{m_1+m_2} \right]$
- i) Fraction of loss in K.E is given by $\frac{\Delta k E}{k E} = \frac{m_2}{m_1+m_2}$
- ii) Percentage of loss in K.E is given by $\frac{\Delta k E}{k E} \times 100 = \left[\frac{m_2}{m_1+m_2} \right] 100$
- iii) Remaining K.E is given by $\Delta k E^1 = E - \Delta k = E \left[\frac{m_1}{m_1+m_2} \right]$
- iv) Fraction of remaining K.E is given by $\frac{\Delta k E^1}{k E} = \frac{m_1}{m_1+m_2}$
- v) Percentage of remaining K.E is given by $\frac{\Delta k E^1}{k E} \times 100 = \frac{m}{m_1+m_2}$
- vi) The remaining K.E is shared between " m_1 " and " m_2 " in the direct ratio of their mass.
- vii) The K.E of m_1 after collision is given by $k E_1^1 = E \left[\frac{m_1}{m_1+m_2} \right]^2$
- viii) The K.E of m_2 after collision $k E_2^1 = E \left[\frac{m_2}{m_1+m_2} \right]^2$

- **Semielastic collision (or) Inelastic collision :** Let a body of mass m_1 moving with the velocity u_1 under goes collision with another body of mass m_2 moving with the velocity u_2 in the same direction. The coefficient of restitution is " e ". The velocity of the bodies after the collision is

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2} \right) u_1 + \left(\frac{m_2 + em_2}{m_1 + m_2} \right) u_2$$

$$v_2 = \left(\frac{m_1 + em_1}{m_1 + m_2} \right) u_1 + \left(\frac{m_2 - em_1}{m_1 + m_2} \right) u_2$$

If the second body is at rest before collision $v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2} \right) u_1$

$$v_2 = \left(\frac{m_1 - em_1}{m_1 + m_2} \right) u_1$$

$$\therefore \frac{v_1}{v_2} = \left(\frac{m_1 - em_2}{m_1 + em_1} \right)$$

- If the mass of the two bodies are equal $V_1 = \left(\frac{1-e}{2} \right) u_1 + \left(\frac{1+e}{2} \right) u_2$

$$V_2 = \left(\frac{1+e}{2} \right) u_1 + \left(\frac{1-e}{2} \right) u_2$$

- If second body is at rest before collision then

$$V_1 = \left(\frac{1-e}{2} \right) u_1; V_2 = \left(\frac{1+e}{2} \right) u_1$$

$$\therefore \frac{V_1}{V_2} = \left(\frac{1-e}{1+e} \right)$$

- If second body is at rest before collision then

$$K.E. = \frac{1}{2} \frac{m_1m_2}{m_1+m_2} (1-e^2) (u_2 - u_1)^2$$

Varaprasad
 Founder/CEO
 The Scholar-Edu-tech
 for IIT/NEET
 Foundation
 6309824365