

The Velocity of the Body after nth Collision?

COLLISIONS

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If second body is at rest before collision then

$$V_1 = \left(\frac{1-e}{2}\right) u_1; V_2 = \left(\frac{1+e}{2}\right) u_1$$

$$\therefore \frac{V_1}{V_2} = \left(\frac{1-e}{1+e}\right)$$

During inelastic collision, there is always loss of kinetic energy which is given by Loss in

$$K.E. = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (1-e^2) (u_2 - u_1)^2$$

Ballistic Pendulum is an example of inelastic collision. When a bullet moving with a velocity 'u' horizontally is fired into the wooden block (suspended by a string) it rises to height 'h' from its mean position. Then mu = (M + m)v

Here $V = \sqrt{2gh}$

i) $V = \frac{(M+m)}{m} \sqrt{2gh}$ Where 'm' is the mass of the bullet. 'M' is the mass of the block.

If the string of the ballistic pendulum makes an angle 'θ' with the vertical after impact and the length of the string is 'l' then velocity of the bullet

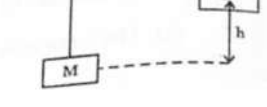
$$u = \frac{(M+m)}{m} \sqrt{2gl(1 - \cos \theta)}$$

Fraction of K.E. lost in this impact = M/(M+m)

iv) Height of the system

$$(h) = \frac{u^2}{2g} \left[\frac{m}{m+M} \right]^2$$

v) $\theta = \cos^{-1} \left[1 - \frac{1}{2gl} \left(\frac{mu}{m+M} \right)^2 \right]$



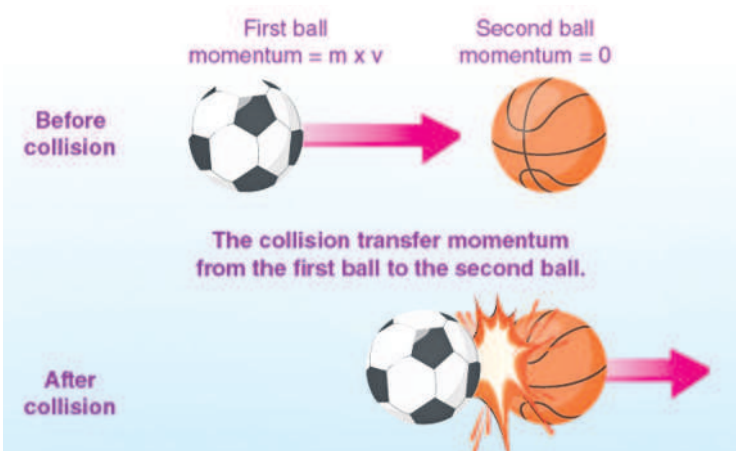
Coefficient of Restitution: During a collision between two bodies, the ratio of the relative velocity of their separation after collision to the relative velocity of their approach before collision is a constant which is called coefficient of restitution (e).

$$\text{coefficient of restitution (e)} = \frac{v_2 - v_1}{u_1 - u_2}$$

(Newton's experimental Law)

Note: The value of 'e' lies between 0 and 1 (For semi-elastic collision).

- For perfectly elastic collisions "e" = 1.
- For perfectly inelastic collisions "e" = 0



If a body falls from a height "h₁" onto hard floor and rebounds to a height "h₂" the coefficient of

restitution is given by $e = \sqrt{\frac{h_2}{h_1}}$

If a body falls from a height 'h' on a horizontal plane, then i) The height through which the body rebounds after "n" collision h_n = e²ⁿ x h.

Where 'e' is the coefficient of restitution.

ii) The velocity of the body after nth collision, v_n = eⁿ x v

iii) The total distance covered by it before it stops rebounding is given by $s = h \left(\frac{1+e^2}{1-e^2} \right)$

iv) The time taken by it come to rest is given by $t = \sqrt{\frac{2h}{g} \left(\frac{1+e}{1-e} \right)}$

Elastic collision in two dimensions :

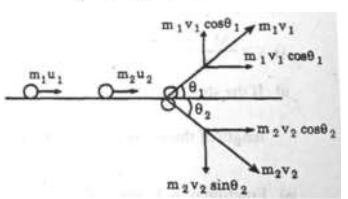
long X - axis,

$$M_1 u_1 + m_2 u_2 = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2$$

long Y - axis,

$$0 = m_1 v_1 \sin \theta_1 - m_2 v_2 \sin \theta_2$$

$$\text{vi) } \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$



Worked Out Examples:

A gun of mass M fires a bullet of mass m. If total energy evolved in firing is E then find momentum of gun Sol: mv = MV = P

∴ Total energy

$$= E = \frac{p^2}{2m} = \frac{p^2}{2M} = \frac{p^2}{2} \left[\frac{m+M}{Mm} \right]$$

∴ momentum of gun, p = $\sqrt{\frac{2MmE}{M+m}}$

A bomb explodes in air when it has horizontal speed of 100km/h. It brakes into two parts A and B of mass ratio 1:2. If A goes vertically up with speed of 400 km/h then

find velocity of B immediately after the explosion.

Sol: $p_B = \sqrt{p^2 + p_A^2}$
 $\therefore \frac{2m}{3} \times v_3$

$$= \sqrt{(100 \times m)^2 + \left(\frac{m}{3} \times 400 \right)^2}$$

∴ V₃ = 250km/h

Two balls each of 0.06 kg mass moving in opposite direction with a velocity 8 m/s collide with each other and move back with equal velocity. Find the change in the momentum of each ball due to the collision.

Sol: m₁ = m₂ 0.006 kg
u₁ = 8 m/sec
u₂ = -8 m/sec

We know that during these type of collisions two bodies simply exchange their velocities.

∴ V₁ = u₂ = -8 m/sec
V₂ = u₁ = 8 m/sec

For 1st body:

Change in momentum = m₁ v₁ - m₁ u₁ = 0.06 x (-8) - 0.06 x 8 = -0.96 kg -m/sec

For 2nd body:

Change in momentum = m₂ v₂ - m₂ u₂ = 0.06 x 8 - 0.06 (-8) = 0.96 kg -m/sec.

A ball of mass 0.4 kg moving with a uniform speed of 2ms⁻¹ strikes a wall normally and rebounds. Treating the collision as elastic and the time of contact of the ball with wall as 0.4s, find the force exerted on the ball.

Sol: Since the collision is elastic, ball rebounds with same velocity (2m/sec.)

∴ Change in momentum of ball = mv - (-mv) = 2mv = 2 x 0.4 x 2 = 1.6 kg-m/sec. Time of contact (t) = 0.4 sec. ∴ Force exerted on the ball (F)

$$\frac{\text{change in momentum}}{\text{time}} = \frac{1.6}{0.4} = 4N$$

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A bullet of mass 5g moving with a velocity of 300 ms⁻¹ strikes a ballistic pendulum of mass 1.995 kg and length 1m and emerges out. Find the velocity and the vertical height through which the pendulum rises.

Sol: m₁ = 5gm = 0.005kg. ; m₂ = 1.995 kg. u₁ = 300 m/sec ; u₂ = 0 Common velocity (v) = ?

i) According to law of conservation of momentum

$$= m_1 u_1 + m_2 u_2 = (m_1 + m_2) V$$

$$\therefore V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2} = \frac{0.005 \times 300 + 1.995 \times 0}{0.005 + 1.995}$$

$$= \frac{1.5}{2} = 0.75 \text{ m/sec}$$

ii) Vertical height reached (h) = ?

According to law of conservation of energy $\frac{1}{2} (m + M) v^2 = (m + M) gh$
 $h = \frac{v^2}{2g} = \frac{0.75 \times 0.75}{2 \times 9.8} = 0.0287 \text{ m}$

A rifle bullet of mass 30 gm leaves the rifle with a velocity of 100m/s the rifle tending to recoil with a velocity of 2 m/s. Find the mass of the rifle.

Sol :Momentum of the rifle = momentum of the bullet

Hence $M \times 2 = \frac{30}{1000} \times 100 = 3$
 $\therefore M = \frac{3}{2} = 1.5 \text{ kg}$

A bullet of mass 'a' travelling with a velocity 'b' strikes a block of wood of mass 'c' which is rest. Find the common velocity after the impact is

Sol : The momentum before impact = ab If v is the common velocity, the momentum after impact.

= av + cv = (a + c) v
 $\therefore (a + c) v = ab$ or, $v = \frac{ab}{a+c}$

A body of mass 20 kg moving with a velocity of 4 m/s collides with another body of mass 10 kg moving with a velocity of 2 m/s moving from the opposite direction. After the collision the two bodies move together, find their common velocity.

Sol : Formula : m₁u₁ - m₂u₂ = v (m₁ + m₂) or, 20 x 4 - 10 x 2 = v (20 + 10) or, 80 - 20 = v x 30 or, 30 v = 60 ∴ v = 2 m/s

A 6 kg box sled is travelling across the ice horizontally at a speed of 9 m/s. When a 12 kg package is dropped into it vertically, what is the subsequent speed of the sled ?

Sol: Momentum of the sled before the package is dropped = 6 x 9

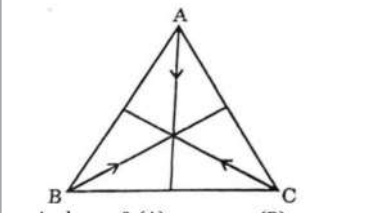
Momentum after the package is dropped = (6+12) v Where v is the common velocity ∴ (6+12) v = 6 x 9 or, 18 v = 6 x 9
 $V = \frac{6 \times 9}{18} = 3 \text{ m/s}$

Three particles A,B and C of equal mass "m" move with equal speed "V" along the medians of an equilateral triangle as shown in Fig. They collide at the centroid "G" of the triangle.



After the collision A comes to rest. B retraces its path with speed V. What is the velocity of C ?

Sol : Since the total momentum before collision is zero, the total momentum after the collision should be zero or, p₁ + p₂ + p₃ = 0



And p₁ = 0 (A) ; p₂ = -mv(B)
 $\therefore -mv + p_3 = 0$ or, p₃ = mv (c)
Hence the body moves with a speed v in the direction B to G and away from G.

Varaprasad
Founder/CEO
The Scholar-Edu-tech
for IIT/NEET
Foundation
6309824365

