

The Velocity of the Body after nth Collision?

COLLISIONS

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- If second body is at rest before collision then

$$V_1 = \left(\frac{1-e}{2}\right) u_1; V_2 = \left(\frac{1+e}{2}\right) u_1$$

$$\therefore \frac{V_1}{V_2} = \left(\frac{1-e}{1+e}\right)$$

- During inelastic collision, there is always loss of kinetic energy which is given by Loss in

$$K.E. = \frac{1}{2} \frac{m_1 m_2}{m_1 + m_2} (1-e^2) (u_2 - u_1)^2$$

- Ballistic Pendulum is an example of inelastic collision. When a bullet moving with a velocity 'u' horizontally is fired into the wooden block (suspended by a string) it rises to height 'h' from its mean position. Then $mu = (M+m)v$
- Here $V = \sqrt{2gh}$

- $V = \frac{(M+m)}{m} \sqrt{2gh}$ Where 'm' is the mass of the bullet. 'M' is the mass of the block.
- If the string of the ballistic pendulum makes an angle 'θ' with the vertical after impact and the length of the string is 'l' then velocity of the bullet

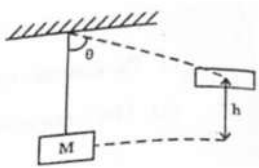
$$u = \frac{(M+m)}{m} \sqrt{2gl(1 - \cos \theta)}$$

- Fraction of K.E. lost in this impact = $M/(M+m)$

- Height of the system

$$(h) = \frac{u^2}{2g} \left[\frac{m}{m+M}\right]^2$$

- $\theta = \cos^{-1} \left[1 - \frac{1}{2gl} \left(\frac{mu}{m+M}\right)^2\right]$

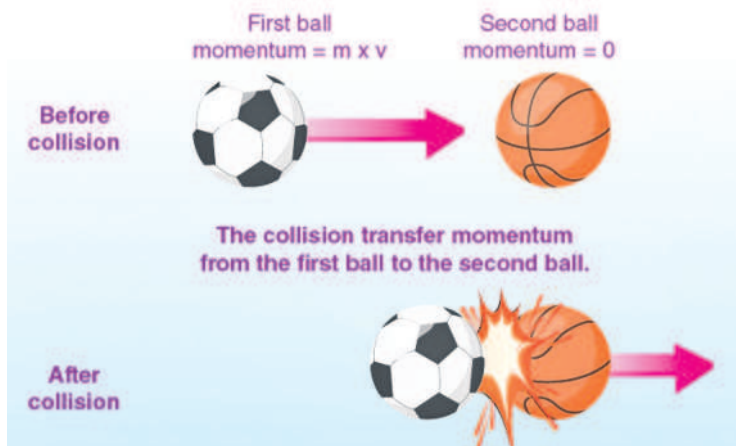


- Coefficient of Restitution: During a collision between two bodies, the ratio of the relative velocity of their separation after collision to the relative velocity of their approach before collision is a constant which is called coefficient of restitution (e).

$$\text{coefficient of restitution (e)} = \frac{v_2 - v_1}{u_1 - u_2}$$

(Newton's experimental Law)

- Note:** The value of 'e' lies between 0 and 1 (For semi-elastic collision).
- For perfectly elastic collisions "e" = 1.
 - For perfectly inelastic collisions "e" = 0



- If a body falls from a height "h₁" onto hard floor and rebounds to a height "h₂" the coefficient of

restitution is given by $e = \sqrt{\frac{h_2}{h_1}}$

- If a body falls from a height 'h' on a horizontal plane, then
- The height through which the body rebounds after "n" collision $h_n = e^{2n} \times h$. Where 'e' is the coefficient of restitution.
 - The velocity of the body after nth collision, $v_n = e^n \times v$
 - The total distance covered by it before it stops rebounding is given by $s = h \left(\frac{1+e^2}{1-e^2}\right)$

- The time taken by it come to rest is given by $t = \sqrt{\frac{2h}{g}} \left(\frac{1+e}{1-e}\right)$

Elastic collision in two dimensions :

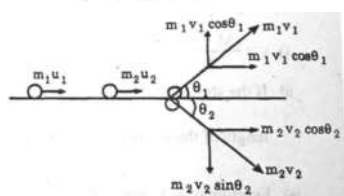
long X - axis,

$$M_1 u_1 + m_2 u_2 = m_1 v_1 \cos \theta_1 + m_2 v_2 \cos \theta_2$$

long Y - axis,

$$0 = m_1 v_1 \sin \theta_1 - m_2 v_2 \sin \theta_2$$

$$vi) \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$



Worked Out Examples:

- A gun of mass M fires a bullet of mass m. If total energy evolved in firing is E then find momentum of gun
- Sol: $mv = MV = P$
- ∴ Total energy
- $$= E = \frac{p^2}{2m} = \frac{p^2}{2M} = \frac{p^2}{2} \left[\frac{m+M}{Mm}\right]$$
- ∴ momentum of gun, $p = \sqrt{\frac{2MmE}{M+m}}$

- A bomb explodes in air when it has horizontal speed of 100km/h. It breaks into two parts A and B of mass ratio 1:2. If A goes vertically up with speed of 400 km/h then

find velocity of B immediately after the explosion.

Sol: $p_B = \sqrt{p^2 + p_A^2}$

$$\therefore \frac{2m}{3} \times v_3$$

$$= \sqrt{(100 \times m)^2 + \left(\frac{m}{3} \times 400\right)^2}$$

$$\therefore V_3 = 250 \text{ km/h}$$

- Two balls each of 0.06 kg mass moving in opposite direction with a velocity 8 m/s collide with each other and move back with equal velocity. Find the change in the momentum of each ball due to the collision.

Sol: $m_1 = m_2 = 0.006 \text{ kg}$

$$u_1 = 8 \text{ m/sec}$$

$$u_2 = -8 \text{ m/sec}$$

We know that during these type of collisions two bodies simply exchange their velocities.

$$\therefore V_1 = u_1 = -8 \text{ m/sec}$$

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For 1st body:

Change in momentum

$$= m_1 v_1 - m_1 u_1 =$$

$$= 0.06 \times (-8) - 0.06 \times 8 =$$

$$= -0.96 \text{ kg -m/sec}$$

For 2nd body:

Change in momentum

$$= m_2 v_2 - m_2 u_2 =$$

$$= 0.06 \times 8 - 0.06 \times (-8) =$$

$$= 0.96 \text{ kg -m/sec.}$$

- A ball of mass 0.4 kg moving with a uniform speed of 2ms⁻¹ strikes a wall normally and rebounds. Treating the collision as elastic and the time of contact of the ball with wall as 0.4s, find the force exerted on the ball.

Sol: Since the collision is elastic, ball rebounds with same velocity (2m/sec.)

∴ Change in momentum of ball = $mv - (-mv) = 2mv$

$$= 2 \times 0.4 \times 2 = 1.6 \text{ kg-m/sec.}$$

Time of contact (t) = 0.4 sec.

∴ Force exerted on the ball (F)

$$\frac{\text{change in momentum}}{\text{time}} = \frac{1.6}{0.4} = 4 \text{ N}$$

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- A bullet of mass 5g moving with a velocity of 300 ms⁻¹ strikes a ballistic pendulum of mass 1.995 kg and length 1m and emerges out. Find the velocity and the vertical height through which the pendulum rises.

Sol: $m_1 = 5 \text{ gm} = 0.005 \text{ kg.}$

$$m_2 = 1.995 \text{ kg.}$$

$$u_1 = 300 \text{ m/sec ; } u_2 = 0$$

Common velocity (v) = ?

- According to law of conservation of momentum

$$= m_1 u_1 + m_2 u_2$$

$$= (m_1 + m_2) V$$

$$\therefore V = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$= \frac{0.005 \times 300 + 1.995 \times 0}{0.005 + 1.995}$$

$$= \frac{1.5}{2}$$

$$= 0.75 \text{ m/sec}$$

- Vertical height reached (h) = ?

According to law of conservation of energy

$$\frac{1}{2} (m + M) v^2 = (m + M) gh$$

$$h = \frac{v^2}{2g} = \frac{0.75 \times 0.75}{2 \times 9.8} = 0.0287 \text{ m}$$

- A rifle bullet of mass 30 gm leaves the rifle with a velocity of 100m/s the rifle tending to recoil with a velocity of 2 m/s. Find the mass of the rifle.

Sol : Momentum of the rifle = momentum of the bullet

$$\text{Hence } M \times 2 = \frac{30}{1000} \times 100 = 3$$

$$\therefore M = \frac{3}{2} = 1.5 \text{ kg}$$

- A bullet of mass 'a' travelling with a velocity 'b' strikes a block of wood of mass 'c' which is rest. Find the common velocity after the impact is

Sol : The momentum before impact = ab If v is the common velocity, the momentum after impact.

$$= av + cv = (a + c) v$$

$$\therefore (a + c) v = ab \text{ or, } v = \frac{ab}{a+c}$$

- A body of mass 20 kg moving with a velocity of 4 m/s collides with another body of mass 10 kg moving with a velocity of 2 m/s moving from the opposite direction. After the collision the two bodies move together, find their common velocity.

Sol : Formula : $m_1 u_1 - m_2 u_2 = v (m_1 + m_2)$ or,

$$20 \times 4 - 10 \times 2 = v (20 + 10)$$

$$80 - 20 = v \times 30 \text{ or,}$$

$$30 v = 60$$

$$\therefore v = 2 \text{ m/s}$$

- A 6 kg box sled is travelling across the ice horizontally at a speed of 9 m/s. When a 12 kg package is dropped into it vertically, what is the subsequent speed of the sled ?

Sol: Momentum of the sled before the package is dropped = 6 x 9

- Momentum after the package is dropped = (6+12) v Where v is the common velocity

$$\therefore (6+12) v = 6 \times 9 \text{ or, } 18 v = 6 \times 9$$

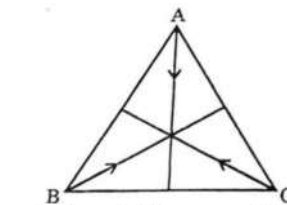
$$V = \frac{6 \times 9}{18} = 3 \text{ m/s}$$

- Three particles A,B and C of equal mass "m" move with equal speed "V" along the medians of an equilateral triangle as shown in Fig. They collide at the centroid "G" of the triangle.



- After the collision A comes to rest. B retraces its path with speed V. What is the velocity of C ?

Sol : Since the total momentum before collision is zero, the total momentum after the collision should be zero or,

$$p_1 + p_2 + p_3 = 0$$


And $p_1 = 0$ (A) ; $p_2 = -mv$ (B)

$$\therefore -mv + p_3 = 0 \text{ or, } p_3 = mv \text{ (c)}$$

Hence the body moves with a speed v in the direction B to G and away from G.

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