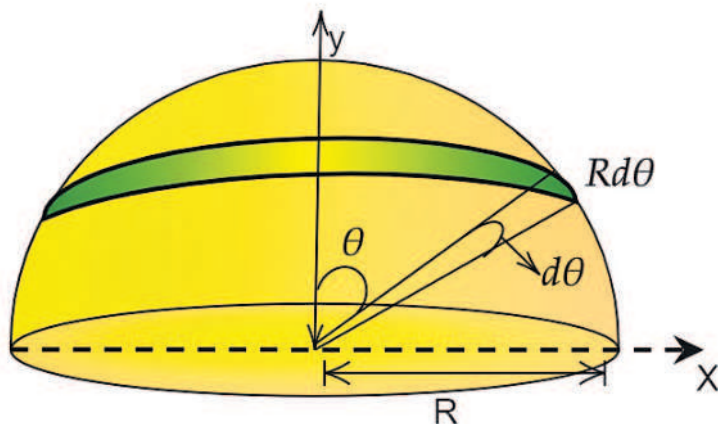


# The motion of the centre of mass depends on..

## Centre of Mass

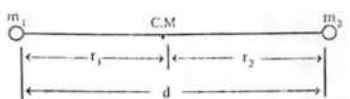
- **Centre of Mass:** The Centre of mass of a body is the point within the boundaries of the body where the total mass of the body appears to be concentrated and when external force acts on the point, the body undergoes translatory motion.
- The motion of Centre of mass of a body represents the motion of the whole body.
- The position of Centre of mass is independent of co-ordinate system and its position remains constant relative to any particles of the system.
- The position of the Centre of mass of a body depends upon the shape of the body, masses of the particles of the body and relative separation between them.
- For symmetrical bodies with homogeneous mass distribution, the Centre of mass coincides with the Centre of symmetry or geometrical Centre.
- The Centre of mass of a body need not be where there is mass. It may lie within the body or outside.

- Ex:** The Centre of mass of a circular disc is at the Centre of the disc (within the body). But the Centre of mass of a circular ring is at its Centre (outside the body).
- For a rigid system, the position of Centre of mass does not vary with time. For a non rigid system, the position of Centre of mass varies with time (**Ex :** solar system).
  - When a body is in translatory motion, the Centre of mass of the body also have translatory motion.
  - When a body is in rotatory motion only, the Centre of mass of the body is at rest.
  - When a body is rolling on a surface (i.e., having translatory and rotatory motion simultaneously) then its Centre of mass has translatory motion only.
  - **Centre of Gravity:** The Centre of gravity of a body is the point where its total weight can be supposed to act.
  - **Centre of gravity is related to weight of all particles of the body.** Note: If the body is small, the Centre of mass and the Centre of gravity



coincide. But if the body is huge (like a mountain) where 'g' is not uniformly distributed, the Centre of mass and the Centre of gravity differ.

- In case of symmetrical bodies the Centre of mass coincide with the geometrical Centre.
- If two particles of masses  $m_1, m_2$  are separated by distance 'd' and  $r_1, r_2$  are the distance of their centre of mass from  $m_1$  and  $m_2$  then  $m_1 r_1 = m_2 r_2$



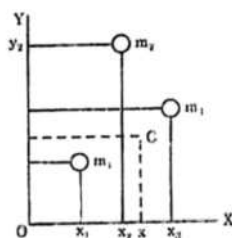
- i.e., The distance of the particles from centre of mass are in the inverse ratio of their masses.
- $\Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1}$
- i) Distance of the centre of mass 'm1' is given by  $r_1 = \left(\frac{d}{m_1+m_2}\right) m_2$
- ii) Distance of the centre of mass from 'm2' is given by  $r_2 = \left(\frac{d}{m_1+m_2}\right) m_1$

- The moments of forces or masses of the system about Centre of mass is equal to zero.
- If  $x_1, x_2$  are the distance of the particles from the origin and XCM is the distance of the centre of mass of the system of the origin,

then  $X_{CM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$   
 For n particle system  
 $X_{CM} = \frac{\sum m_i x_i}{M}$

- The motion of the Centre of the mass depends on the mass depends on the external forces acting on the system.
- The motion of the centre of mass does not depend on the internal forces acting on the

- system.
- Centre of mass for a System of Particles: Let a system of particles of masses  $m_1, m_2, m_3, \dots$  lie in the X-Y plane with respective co-ordinate  $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ . And x and y are the co-ordinates of their centre of mass C.



$$X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} \dots\dots(1)$$

And

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \dots\dots(2)$$

- Let a system of particles of masses  $m_1, m_2, m_3 \dots$  Lie in space with co-ordinates  $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) \dots$  then the co-ordinates of C.M. of the system are

And  
 $Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} \dots\dots(2)$

Let a system of particles of masses  $m_1, m_2, m_3 \dots$  Lie in space with co-ordinates  $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3) \dots$  then the co-ordinates of C.M. of the system are

$$X_{CM} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

$$Y_{CM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

$$Z_{CM} = \frac{m_1 z_1 + m_2 z_2 + m_3 z_3 + \dots}{m_1 + m_2 + m_3 + \dots}$$

If r is the distance of the centre of mass from the origin of the coordinate system, then  $r = \sqrt{X_{CM}^2 + Y_{CM}^2 + Z_{CM}^2}$

if  $\vec{r}_1$  and  $\vec{r}_2$  are the position vectors of the particles having masses  $m_1$  and  $m_2$  then the

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position vectors of the centre of mass of the system is

$$\vec{r}_{CM} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2}$$

For a rigid body having continuous mass distribution.

$$X_{CM} = \frac{\int x dm}{\int dm}; Y_{CM} = \frac{\int y dm}{\int dm} \text{ and}$$

$$Z_{CM} = \frac{\int z dm}{\int dm}.$$

- When external forces act on system become zero, then the centre of mass of the system may be either at rest or in uniform motion in a straight line.

**Ex:**

- If a bomb at rest explodes into no. of pieces the centre of mass remains stationary since forces involved are action - reaction pairs which are internal forces.
- A bomb is dropped freely from certain height. After some time exploded into no. of pieces. Then the path of the centre of mass of the bomb after explosion is a vertical straight line.
- If a shell is projected at an angle with the horizontal explodes in mid air, the centre of mass continues to move with same parabolic path till anyone fragment reaches the ground.
- Two bodies of masses  $m_1$  and  $m_2$  are separated by a distance d and initially they are at rest. Now they move towards each other under mutual force of attraction. The velocity of centre of mass when they collided is zero.
- If two bodies coupled by a string are pulled apart and released. The velocity of centre of mass is zero.
- A boat is floating on water surface. A man travels from one end of the boat to its other end. Then the centre of mass of the system is unchanged since the forces involved are internal forces.
- A person of mass m is standing on a boat of mass M so that he is d metre from shore. If he walks / metre on boat towards the shore then
  - The distance travelled by the boat in opposite direction

$$(d^1) = \frac{ml}{M+m}$$

b) The distance of person from shore =  $d + d^1 - l$ .

- The motion of centre of mass can be determined using Newton's second law,  $F = m \vec{a}$

### Velocity of Centre of mass:

- If different particles of masses  $m_1, m_2, m_3 \dots m_n$  move with velocities  $\vec{v}_1, \vec{v}_2, \vec{v}_3 \dots \vec{v}_n$  respectively, then the velocity of the centre of mass is given by

$$\vec{V}_{CM} = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \dots + m_n \vec{v}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$= \frac{\sum m \vec{v}}{\sum m} \text{ or } \vec{V}_{CM} = \frac{\vec{P}}{M}$$

- Where P is total momentum of 'n' particles 'M' is total mass of 'n' particles.

**Note:** The velocity of centre of mass remains constant when no external force acts on the system.

- When two particles of masses  $m_1$  and  $m_2$  are moving at right angles to each other with velocities  $v_1$  and  $v_2$ , then the velocity of their centre of mass is given by

$$V_{CM} = \sqrt{\frac{m_1^2 v_1^2 + m_2^2 v_2^2}{(m_1 + m_2)^2}}$$

- (a) If  $v_1$  and  $v_2$  are the velocities of particles of masses  $m_1$  and  $m_2$  moving in the same direction then the velocity of centre of mass of the system is

$$V_{CM} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

- (b) If they move in opposite direction then  $V_{CM} = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2}$

The total momentum of all the particles with respect to the centre of mass is zero.

### Acceleration of Centre of mass:

- If different particles of masses  $m_1, m_2, m_3 \dots m_n$  move with acceleration  $\vec{a}_1, \vec{a}_2, \vec{a}_3 \dots \vec{a}_n$  respectively, then the acceleration of the centre of mass is given by

$$a_{CM} = \frac{m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 + \dots + m_n \vec{a}_n}{m_1 + m_2 + m_3 + \dots + m_n}$$

$$= \frac{\sum m \vec{a}}{\sum m}$$

$$\vec{F}_{\text{external}} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_n = M \vec{a}_{CM}$$

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