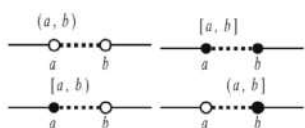


The collection of all the subsets of the set is called ?

continued from Oct 23rd

Defines the set of negative real numbers. The set $(-\infty, \infty)$ describes the set of real numbers in relation to a line extending from $-\infty$ to ∞ . On real number line, various types of intervals described above as subsets of \mathbb{R} , are shown in the following figure :



- Here, we note that an interval contains infinitely many points.
- For example, the set $\{x : x \in \mathbb{R}, -5 < x \leq 7\}$, written in set-builder form, can be written in interval as $(-5, 7]$ and the interval $[-3, 5)$ can be written in set-builder form as $\{x : -3 \leq x < 5\}$.
- The number $(b - a)$ is called the length of any of the intervals (a, b) , $[a, b]$, $(a, b]$ or $[a, b)$. The intervals $(a, b]$ and $[a, b)$ are also referred as semi-closed (or semi-open) intervals.

POWER SET

Let A is a given set. The collection of all the subsets of the set A is called the power set of A . It is denoted by $P(A)$.

Hence, $P(A) = \{S : S \subseteq A\}$

For example :

If $A = \{a, b, c\}$ then $P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a, b, c\}\}$

Clearly if A is a finite set and $n(A) = m$, then $n(P(A)) = 2^m$

UNIVERSAL SET

A set, which contains all sets under consideration as subsets is called the universal set. It is denoted by U . The choice of universal set is not unique. Different universal sets are used in different contexts.

COMPARABLE SETS

Two sets A and B are said to be comparable if

$$A \subseteq B \text{ or } B \subseteq A$$

VENN DIAGRAM

Most of the relationship between sets can be represented by means of diagrams called Venn diagrams. In the Venn diagram the universal set U is represented by the interior of a rectangle. Other sets under consideration are represented

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by the interior of circles drawn inside the rectangle.

- If a set A is a subset of a set B then the circle representing A is drawn inside the circle representing B .

OPERATIONS ON SETS

UNION OF SETS

- Let A and B be two sets. The union of A and B is the set of all those elements which belong to A or B or A and B both. Symbolically we write $A \cup B$ which is read as "A union B" Thus
- In the Venn diagrams the

$$A \cup B = \{x : x \in A \text{ or } x \in B\} \text{ or } x \in A \cup B \text{ or } x \in A \text{ or } x \in B$$

Also, $x \in A \cup B \Rightarrow x \in A \text{ and } x \in B$

shaded regions represent the union of sets A and B in different cases



- The union of a number of sets $A_1, A_2, A_3, \dots, A_n$, i.e. $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$ is represented by

For example :

- If $A = \{1,2,3\}$; $B = \{2,3,4,5\}$. Then $A \cup B = \{1,2,3,4,5\}$
- If $A = \{a, e, i, o, u\}$; $B = \{e, o, u\}$. Then $A \cup B = \{a, e, i, o, u\}$
- If $A = \{1,2\}$; $B = \{a,b,c\}$ Then $A \cup B = \{1,2,a,b,c\}$
- If $A = \{x : x \in \mathbb{I}^+\}$; $B = \{x : x \in \mathbb{I}^-\}$ and $x < 0$;
- Then $A \cup B = \{ \dots, -4, -3, -2, -1, -1, -2, -3, \dots \}$

ALGEBRA OF UNION

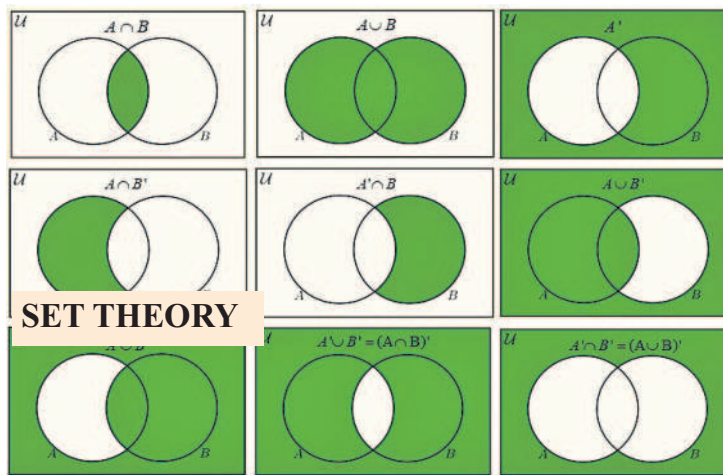
- Let A, B, C be any three sets defined in the universal set U , then

- Idempotent Law: $A \cup A = A$
- Commutative Law : $A \cup B = B \cup A$
- Associative Law : $(A \cup B) \cup C = A \cup (B \cup C)$
- Identity Law : (i) $A \cup \emptyset = A$, (ii) $A \cup U = U$

INTERSECTION OF SETS

- Let A and B are two sets. The intersection of the sets A and B is the set of all those elements which belong to both A and B . Symbolically, we write $A \cap B$, which is read as "A intersection B" Thus, $A \cap B = \{x : x \in A \text{ and } x \in B\}$ or $x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$

Also, $x \in A \cap B \Rightarrow x \in A \text{ or } x \in B$



In the following Venn diagrams, the shaded regions represent the intersection of sets A and B in different cases.



- The intersection of a number of sets $A_1, A_2, A_3, \dots, A_n$, i.e. $A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n$ is represented by $\bigcap_{i=1}^n A_i$

For example :

- If $A = \{2,4,7,10\}$ and $B = \{1, 2, 3, 4\}$, Then $A \cap B = \{2,4\}$
- If $A = \{x : x \text{ is a prime number}\}$ and $B = \{x : x \in \mathbb{N}\}$
- Then $A \cap B = \{x : x \text{ is a prime number}\} = A$. [Note that $A \subseteq B$]
- If $A = \{1,3,5,7,9,\dots\}$; $B = \{2,4,6,8,\dots\}$, Then $A \cap B = \emptyset$.

ALGEBRA OF INTERSECTION

- Idempotent Law : $A \cap A = A$
- Commutative Law : $A \cap B = B \cap A$
- Associative Law : $(A \cap B) \cap C = A \cap (B \cap C)$
- Identity Law : (i) $A \cap \emptyset = \emptyset$, (ii) $A \cap U = A$
- Distributive law : (i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

FUNCTIONS

INTRODUCTION

- The function has already been defined in the chapter of "Sets, Relations and Functions" as a special kind of relation. However, we rarely use a function in that form. We will define a function in slightly different way as following.
- Let A and B be two sets. Let 'f' is a rule which associates to each member of A a member of B then 'f' is said to be a function from A into B . Symbolically, we write.

The set A is said to be $f: A \rightarrow B$ or $A \xrightarrow{f} B$

DOMAIN and the set B is said to be CO-DOMAIN of the function f .

- If x denotes a member of the set A and y is a member of the set B , which the function f associates to x , then we write $y = f(x)$
- y is called the IMAGE of x under the function f
- x is called the PER-IMAGE of y .
- The set of all the images (or function values) is called RANGE of f . Thus Range of $f = \{f(x) : x \in A\} \subseteq B$. Range of f is also denoted by $f(A)$.
- If $y = f(x)$ is a function of x such that for every value of x in its domain, there corresponds a unique value of y , then $y = f(x)$ is said to be a SINGLE VALUED FUNCTION

of x . In general the function wherever used onwards always means single valued function which is identical to the function defined in relations.

(a single valued function), then Hence $f: A \rightarrow B$, If f is a function

- Each element of A must have exactly one image in B , i.e., no element of A
- should be without an image and no element of A can have more than one image.
- Two or more element in A may have same image in B .
- There may be some elements in B which are not images of elements of A .

INDEPENDENT AND DEPENDENT VARIABLES

- The symbol which denotes a member of the domain of function (usually denoted by x) is called an independent

variable and the symbol denoting member of the range of function (usually denoted by y) is called a dependent variable.

REAL VALUED FUNCTION (OR REAL FUNCTION)

- The function for which the domain and range are the subsets of the set of real number \mathbb{R} is called a real function.

WAYS OF DEFINING A REAL FUNCTION

UNIFORM DEFINITION

- If a function is defined as $y = f(x)$, $x \in [a, b]$, we say that it is uniformly defined. For example :

- $y = f(x) = \sin x$, $x \in \mathbb{R}$
 - $y = f(x) = x^2 + 1$, $x \in [-1, 1]$
- #### PIECEWISE DEFINITION
- If a function $y = f(x)$, $x \in [a, b]$ assumes different forms in different subsets of $[a, b]$, we say that it is piecewise defined. For example :

$$y = f(x) = \begin{cases} 1, & -1 \leq x \leq 0 \\ 1-x, & 0 \leq x < 1 \\ x-1, & x \geq 1 \end{cases}$$

$$y = f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$$

EXPLICIT AND IMPLICIT FUNCTIONS

- EXPLICIT FUNCTIONS A function is said to be an explicit function if it is expressed in the form $y = f(x)$. That is the dependent variable y is expressible completely in terms of the independent variable x . For example :

$$y = \log x, x > 0$$

$$y = 3e^x - x^2 + 2, x \in \mathbb{R}$$

IMPLICIT FUNCTIONS

- A function is said to be an implicit function if it is expressed in the form $f(x, y) = C$, where C is a constant. That is y is not directly expressible in terms of x . For example :

$$\sin(x+y) - \cos(x+y) = x+2$$

$$ye^{x^2} + xe^{y^2} = e^{xy}$$

Note : A function given in the implicit form may be reduced (though not always) to an explicit form. Further, most of the implicit functions are not single valued.

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