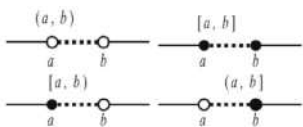


# The collection of all the subsets of the set is called ?

continued from Oct 23<sup>rd</sup>

Defines the set of negative real numbers. The set  $(-\infty, \infty)$  describes the set of real numbers in relation to a line extending from  $-\infty$  to  $\infty$ . On real number line, various types of intervals described above as subsets of  $\mathbb{R}$ , are shown in the following figure :



- Here, we note that an interval contains infinitely many points.
- For example, the set  $\{x : x \in \mathbb{R}, -5 < x \leq 7\}$ , written in set-builder form, can be written in interval as  $(-5, 7]$  and the interval  $[-3, 5)$  can be written in set-builder form as  $\{x : -3 \leq x < 5\}$ .
- The number  $(b - a)$  is called the length of any of the intervals  $(a, b)$ ,  $[a, b]$ ,  $(a, b]$  or  $[a, b)$ . The intervals  $(a, b]$  and  $[a, b)$  are also referred as semi-closed (or semi-open) intervals.

### POWER SET

Let  $A$  is a given set. The collection of all the subsets of the set  $A$  is called the power set of  $A$ . It is denoted by  $P(A)$ .

Hence,  $P(A) = \{S : S \subseteq A\}$

### For example :

If  $A = \{a, b, c\}$  then  $P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}, \{c,a\}, \{a, b, c\}\}$

Clearly if  $A$  is a finite set and  $n(A) = m$ , then  $n(P(A)) = 2^m$

### UNIVERSAL SET

A set, which contains all sets under consideration as subsets is called the universal set. It is denoted by  $U$ . The choice of universal set is not unique. Different universal sets are used in different contexts.

### COMPARABLE SETS

Two sets  $A$  and  $B$  are said to be comparable if

$$A \subseteq B \text{ or } B \subseteq A$$

### VENN DIAGRAM

Most of the relationship between sets can be represented by means of diagrams called Venn diagrams. In the Venn diagram the universal set  $U$  is represented by the interior of a rectangle. Other sets under consideration are represented

## MATHEMATICS

### IIT/NEET Foundation

by the interior of circles drawn inside the rectangle.

- If a set  $A$  is a subset of a set  $B$  then the circle representing  $A$  is drawn inside the circle representing  $B$ .

### OPERATIONS ON SETS

#### UNION OF SETS

- Let  $A$  and  $B$  be two sets. The union of  $A$  and  $B$  is the set of all those elements which belong to  $A$  or  $B$  or  $A$  and  $B$  both. Symbolically we write  $A \cup B$  which is read as "A union B" Thus
- In the Venn diagrams the

$$A \cup B = \{x : x \in A \text{ or } x \in B\} \text{ or } x \in A \cup B \text{ or } x \in A \text{ or } x \in B$$

Also,  $x \in A \cup B \Rightarrow x \in A \text{ and } x \in B$

shaded regions represent the union of sets  $A$  and  $B$  in different cases



- The union of a number of sets  $A_1, A_2, A_3, \dots, A_n$ , i.e.  $A_1 \cup A_2 \cup A_3 \cup \dots \cup A_n$  is represented by

### For example :

- If  $A = \{1,2,3\}$ ;  $B = \{2,3,4,5\}$ . Then  $A \cup B = \{1,2,3,4,5\}$
- If  $A = \{a, e, i, o, u\}$ ;  $B = \{e, o, u\}$ . Then  $A \cup B = \{a, e, i, o, u\}$
- If  $A = \{1,2\}$ ;  $B = \{a,b,c\}$  Then  $A \cup B = \{1,2,a,b,c\}$
- If  $A = \{x : x \in \mathbb{I}^+\}$ ;  $B = \{x : x \in \mathbb{I}^-\}$  and  $x < 0$  ;
- Then  $A \cup B = \{ \dots, -4, -3, -2, -1, -1, -2, -3, \dots \} = \mathbb{I}$

### ALGEBRA OF UNION

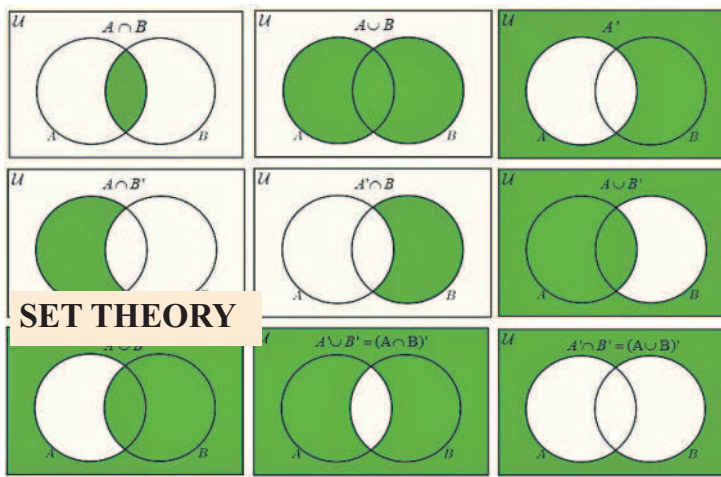
- Let  $A, B, C$  be any three sets defined in the universal set  $U$ , then

- Idempotent Law:  $A \cup A = A$
- Commutative Law :  $A \cup B = B \cup A$
- Associative Law :  $(A \cup B) \cup C = A \cup (B \cup C)$
- Identity Law : (i)  $A \cup \phi = A$ , (ii)  $A \cup U = U$

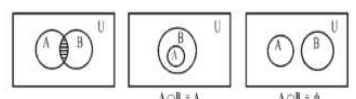
### INTERSECTION OF SETS

- Let  $A$  and  $B$  are two sets. The intersection of the sets  $A$  and  $B$  is the set of all those elements which belong to both  $A$  and  $B$ . Symbolically, we write  $A \cap B$ , which is read as "A intersection B" Thus,  $A \cap B = \{x : x \in A \text{ and } x \in B\}$  or  $x \in A \cap B \Rightarrow x \in A \text{ and } x \in B$

Also,  $x \in A \cap B \Rightarrow x \in A \text{ or } x \in B$



- In the following Venn diagrams, the shaded regions represent the intersection of sets  $A$  and  $B$  in different cases.



- The intersection of a number of sets  $A_1, A_2, A_3, \dots, A_n$ , i.e.  $A_1 \cap A_2 \cap A_3 \cap \dots \cap A_n$  is represented by  $\bigcap_{i=1}^n A_i$

### For example :

- If  $A = \{2,4,7,10\}$  and  $B = \{1, 2, 3, 4\}$ , Then  $A \cap B = \{2,4\}$
- If  $A = \{x : x \text{ is a prime number}\}$  and  $B = \{x : x \in \mathbb{N}\}$
- Then  $A \cap B = \{x : x \text{ is a prime number}\} = A$ . [Note that  $A \subseteq B$ ]
- If  $A = \{1,3,5,7,9,\dots\}$ ;  $B = \{2,4,6,8,\dots\}$ , Then  $A \cap B = \phi$ .

### ALGEBRA OF INTERSECTION

- Idempotent Law :  $A \cap A = A$
- Commutative Law :  $A \cap B = B \cap A$
- Associative Law :  $(A \cap B) \cap C = A \cap (B \cap C)$
- Identity Law : (i)  $A \cap \phi = \phi$ , (ii)  $A \cap U = A$
- Distributive law : (i)  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$  (ii)  $A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$

## FUNCTIONS

### INTRODUCTION

- The function has already been defined in the chapter of "Sets, Relations and Functions" as a special kind of relation. However, we rarely use a function in that form. We will define a function in slightly different way as following.
- Let  $A$  and  $B$  be two sets. Let 'f' is a rule which associates to each member of  $A$  a member of  $B$  then 'f' is said to be a function from  $A$  into  $B$ . Symbolically, we write.

The set  $A$  is said to be  $f: A \rightarrow B$  or  $A \xrightarrow{f} B$

DOMAIN and the set  $B$  is said to be CO-DOMAIN of the function  $f$ .

- If  $x$  denotes a member of the set  $A$  and  $y$  is a member of the set  $B$ , which the function  $f$  associates to  $x$ , then we write  $y = f(x)$
- $y$  is called the IMAGE of  $x$  under the function  $f$
- $x$  is called the PER-IMAGE of  $y$ .
- The set of all the images (or function values) is called RANGE of  $f$ . Thus Range of  $f = \{f(x) : x \in A\} \subseteq B$ . Range of  $f$  is also denoted by  $f(A)$ .
- If  $y = f(x)$  is a function of  $x$  such that for every value of  $x$  in its domain, there corresponds a unique value of  $y$ , then  $y = f(x)$  is said to be a SINGLE VALUED FUNCTION

- of  $x$ . In general the function wherever used onwards always means single valued function which is identical to the function defined in relations.
- (a single valued function), then

Hence  $f: A \rightarrow B$ , If  $f$  is a function

- Each element of  $A$  must have exactly one image in  $B$ , i.e., no element of  $A$
- should be without an image and no element of  $A$  can have more than one image.
- Two or more element in  $A$  may have same image in  $B$ .
- There may be some elements in  $B$  which are not images of elements of  $A$ .

### INDEPENDENT AND DEPENDENT VARIABLES

- The symbol which denotes a member of the domain of function (usually denoted by  $x$ ) is called an independent

variable and the symbol denoting member of the range of function (usually denoted by  $y$ ) is called a dependent variable.

### REAL VALUED FUNCTION (OR REAL FUNCTION)

- The function for which the domain and range are the subsets of the set of real number  $\mathbb{R}$  is called a real function.

### WAYS OF DEFINING A REAL FUNCTION

#### UNIFORM DEFINITION

- If a function is defined as  $y = f(x)$ ,  $x \in [a, b]$ , we say that it is uniformly defined. For example :

- $y = f(x) = \sin x$ ,  $x \in \mathbb{R}$
- $y = f(x) = x^2 + 1$ ,  $x \in [-1, 1]$

#### PIECEWISE DEFINITION

- If a function  $y = f(x)$ ,  $x \in [a, b]$  assumes different forms in different subsets of  $[a, b]$ , we say that it is piecewise defined. For example :

$$y = f(x) = \begin{cases} 1, & -1 \leq x \leq 0 \\ 1-x, & 0 \leq x < 1 \\ x-1, & x \geq 1 \end{cases}$$

$$y = f(x) = \begin{cases} 1, & \text{if } x \text{ is rational} \\ 1-x, & \text{if } x \text{ is irrational} \end{cases}$$

### EXPLICIT AND IMPLICIT FUNCTIONS

- EXPLICIT FUNCTIONS A function is said to be an explicit function if it is expressed in the form  $y = f(x)$ . That is the dependent variable  $y$  is expressible completely in terms of the independent variable  $x$ . For example :

$$y = \log x, x > 0$$

$$y = 3e^x - x^2 + 2, x \in \mathbb{R}$$

### IMPLICIT FUNCTIONS

- A function is said to be an implicit function if it is expressed in the form  $f(x, y) = C$ , where  $C$  is a constant. That is  $y$  is not directly expressible in terms of  $x$ . For example :

$$\sin(x+y) - \cos(x+y) = x+2$$

$$ye^{x^2} + xe^{y^2} = e^{xy}$$

Note : A function given in the implicit form may be reduced (though not always) to an explicit form. Further, most of the implicit functions are not single valued.

**Varaprasad**  
 Founder/CEO  
 The Scholar-Edu-tech  
 for IIT/NEET  
 Foundation  
 6309824365